

Single-parton scattering versus double-parton scattering in the production of two $c\bar{c}$ pairs and charmed meson correlations at the LHC

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Abstract

We compare results of exact calculations of single parton scattering (SPS) and double parton scattering (DPS) for production of $c\bar{c}c\bar{c}$ and for D meson correlations. The SPS calculations are performed in collinear approximation with exact matrix element for $gg \rightarrow c\bar{c}c\bar{c}$ and $q\bar{q} \rightarrow c\bar{c}c\bar{c}$ subprocesses. It is shown that the contribution of gluon-gluon subprocess is about factor 50 larger than that for quark-antiquark annihilation. The new results are compared with results of previous calculation with the approximate matrix element for $gg \rightarrow c\bar{c}c\bar{c}$ in the high-energy approximation. The cross section for the present exact calculation is bigger only at small invariant masses and small rapidity difference between two c quarks (or two \bar{c} antiquarks). We compare correlations in rapidities of two c (or two \bar{c}) for DPS and SPS contributions. Finally we compare our predictions for D mesons with recent results of the LHCb collaboration for invariant mass, rapidity distance between mesons and dimeson invariant mass. The predicted shapes are similar to the measured ones, however, some strength seems to be lacking. Our new calculations clearly confirm the dominance of DPS in the production of events with double charm.

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I. INTRODUCTION

It was advocated in Ref. [1] that the cross section for $c\bar{c}c\bar{c}$ production at high energy may be very large due to double parton scattering (see also [2]). The first evaluation was performed in leading order in collinear approximation. In the meantime the LHCb collaboration measured the cross section for the production of $D^0\bar{D}^0$ pairs at $\sqrt{s} = 7$ TeV which is surprisingly large [3]. Some interesting differential studies were performed there.

Somewhat later two of us discussed several differential distributions for double charm production in the k_t -factorization approach using unintegrated gluon distributions [4] and found several observables useful to identify the DPS effects. So far the single parton scattering contribution to $c\bar{c}c\bar{c}$ was calculated only in high-energy approximation [5]. For kinematical reasons the approximation should be reasonable for large rapidity distances. In real experiments the condition of large rapidity distances is not always fulfilled.

The double scattering effects were studied in several other processes such as four jet production [6–8], production of W^+W^- pairs [9, 10], production of four charged leptons [11–13]. In all the cases the DPS contributions are much smaller than the SPS contributions. The production of double hidden charm was studied e.g. in Ref. [14] for the $pp \rightarrow J/\psi J/\psi X$ process. There the SPS and DPS contributions are comparable. The DPS contribution exceeds the SPS contribution for large rapidity distance between the two J/ψ 's. This is similar for $c\bar{c}c\bar{c}$ production [4].

The aim of the present study is to make a detailed comparison of the DPS and SPS with full (exact) matrix element. The results presented here are obtained with a code which automatically generates matrix element. In the present study we include only $gg \rightarrow c\bar{c}c\bar{c}$ subprocess which is sufficient at high energies. We intend to make a detailed comparison of the present SPS results to the results obtained in the high-energy approximation [5]. Finally our results will be compared to recent LHCb data [3].

II. SOME DETAILS OF THE CALCULATION

In leading-order collinear approximation the differential distributions for $c\bar{c}$ production depend e.g. on the rapidity of the quark, the rapidity of the antiquark and the transverse momentum of one of them (they are identical) [1]. In the next-to-leading order (NLO) collinear approach or in the k_t -factorization approach the situation is more complicated as there are more kinematical variables necessary to describe the kinematical situation. In the k_t -factorization approach the differential cross section for DPS production of $c\bar{c}c\bar{c}$ system, assuming factorization of the DPSmodel, can be written as:

$$\frac{d\sigma^{DPS}(pp \rightarrow c\bar{c}c\bar{c}X)}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t} dy_3 dy_4 d^2p_{3,t} d^2p_{4,t}} = \frac{1}{2\sigma_{eff}} \cdot \frac{d\sigma^{SPS}(pp \rightarrow c\bar{c}X_1)}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} \cdot \frac{d\sigma^{SPS}(pp \rightarrow c\bar{c}X_2)}{dy_3 dy_4 d^2p_{3,t} d^2p_{4,t}}. \quad (2.1)$$

When integrating over kinematical variables one obtains

$$\sigma^{DPS}(pp \rightarrow c\bar{c}c\bar{c}X) = \frac{1}{2\sigma_{eff}} \sigma^{SPS}(pp \rightarrow c\bar{c}X_1) \cdot \sigma^{SPS}(pp \rightarrow c\bar{c}X_2). \quad (2.2)$$

These formulae assume that the two parton subprocesses are not correlated. The parameter σ_{eff} in the denominator of above formulae can be defined in the impact parameter space as:

$$\sigma_{eff} = \left[\int d^2b (T(\vec{b}))^2 \right]^{-1}, \quad (2.3)$$

where the overlap function

$$T(\vec{b}) = \int f(\vec{b}_1) f(\vec{b}_1 - \vec{b}) d^2b_1, \quad (2.4)$$

of the impact-parameter dependent double-parton distributions (dPDFs) are written in the following factorized approximation [15, 16]:

$$\Gamma_{i,j}(x_1, x_2; \vec{b}_1, \vec{b}_2; \mu_1^2, \mu_2^2) = F_{i,j}(x_1, x_2; \mu_1^2, \mu_2^2) f(\vec{b}_1) f(\vec{b}_2). \quad (2.5)$$

Then the impact-parameter distribution can be written as

$$\Gamma(b, x_1, x_2; \mu_1^2, \mu_2^2) = F(x_1, \mu_1^2) F(x_2, \mu_2^2) F(b; x_1, x_2, \mu_1^2, \mu_2^2), \quad (2.6)$$

where b is the parton separation in the impact parameters space. In the formula above the function $F(b; x_1, x_2, \mu_1^2, \mu_2^2)$ contains all information about correlations between the two partons (two gluons in our case). The dependence was studied numerically in Ref. [16] within the Lund Dipole Cascade model. The biggest discrepancy was found in the small b region, particularly for large μ_1^2 and/or μ_2^2 . In general, the effective cross section may depend on kinematical variables:

$$\sigma_{eff}(x_1, x_2, x'_1, x'_2, \mu_1^2, \mu_2^2) = \left(\int d^2b F(b; x_1, x_2, \mu_1^2, \mu_2^2) F(b; x'_1, x'_2, \mu_1^2, \mu_2^2) \right)^{-1}. \quad (2.7)$$

The effect discussed in Ref. [16] may give $\sim 10 - 20$ % on integrated cross section and even more $\sim 30 - 50$ % in some particular parts of the phase space.

In the present study we concentrate, however, rather on higher-order corrections and ignore the interesting dependence of the impact factors on kinematical variables. The dependence may be different for different dynamical models used.

Gaunt and Stirling [15] also ignored the dependence of the impact factors, but included the evolution of the double-parton distribution amplitudes. In our previous paper [1] we have shown that the evolution has very small impact on the cross section for $pp \rightarrow c\bar{c}c\bar{c}X$.

Experimental data from Tevatron [17] and LHC [3, 18, 19] provide an estimate of σ_{eff} in the denominator of formula (2.2). A detailed analysis of σ_{eff} based on the experimental data can be found in Ref. [20, 21]

In our analysis we take $\sigma_{eff} = 15$ mb. In the most general case one may expect some violation of this simple factorized Ansatz given by Eq. 2.2 [16].

In the present approach we concentrate on high energies and therefore ignore the quark induced processes. They could be important only at extremely large pseudorapidities, very large transverse momenta and huge $c\bar{c}$ invariant masses. In the present analysis we avoid these regions of phase space.

In our present analysis cross section for each step is calculated in the k_t -factorization approach (as in Ref. [22]), that is:

$$\begin{aligned} \frac{d\sigma^{SPS}(pp \rightarrow c\bar{c}X)}{dy_c dy_{\bar{c}} d^2p_{c,t} d^2p_{\bar{c},t}} &= \frac{1}{16\pi^2 \hat{s}^2} \int \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi} \overline{|\mathcal{M}_{g_1^* g_2^* \rightarrow c\bar{c}}|^2} \\ &\times \delta^2(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{c,t} - \vec{p}_{\bar{c},t}) \mathcal{F}(x_1, k_{1t}^2, \mu^2) \mathcal{F}(x_2, k_{2t}^2, \mu^2), \end{aligned}$$

The matrix elements for $g^* g^* \rightarrow c\bar{c}$ (off-shell gluons) must be calculated including transverse momenta of initial gluons as it was done first in Refs. [23–25]. The unintegrated (k_t -dependent) gluon distributions (UGDFs) in the proton are taken from the literature [26–28]. Due to the emission of extra gluons encoded in these objects, it is believed that a sizeable part of NLO corrections is effectively included. The framework of the k_t -factorization approach is often used with success in describing inclusive spectra of D or B mesons as well as for theoretical predictions for so-called nonphotonic leptons, products of semileptonic decays of charm and bottom mesons [29–35].

Now we go to the SPS production mechanisms of $c\bar{c}c\bar{c}$. The elementary cross section for the SPS mechanism of double $c\bar{c}$ production has the following generic form:

$$d\hat{\sigma} = \frac{1}{2\hat{s}} \overline{|\mathcal{M}_{gg \rightarrow c\bar{c}c\bar{c}}|^2} d^4PS. \quad (2.8)$$

where

$$d^4PS = \frac{d^3p_1}{E_1(2\pi)^3} \frac{d^3p_2}{E_2(2\pi)^3} \frac{d^3p_3}{E_3(2\pi)^3} \frac{d^3p_4}{E_4(2\pi)^3} \delta^4(p_1 + p_2 + p_3 + p_4). \quad (2.9)$$

Above p_1, p_2, p_3, p_4 are four-momenta of final c, \bar{c}, c, \bar{c} quarks and antiquarks, respectively.

Neglecting small electroweak corrections, the hadronic cross section is then the integral

$$\begin{aligned} d\sigma &= \int dx_1 dx_2 (g(x_1, \mu_F^2) g(x_2, \mu_F^2) d\sigma_{gg \rightarrow c\bar{c}c\bar{c}} \\ &+ \sum_f q_f(x_1, \mu_F^2) \bar{q}_f(x_2, \mu_F^2) d\sigma_{q\bar{q} \rightarrow c\bar{c}c\bar{c}} \\ &+ \sum_f \bar{q}_f(x_1, \mu_F^2) q_f(x_2, \mu_F^2) d\sigma_{\bar{q}q \rightarrow c\bar{c}c\bar{c}}). \end{aligned} \quad (2.10)$$

In the calculation below we include $u\bar{u}, \bar{u}u, d\bar{d}, \bar{d}d, s\bar{s}, \bar{s}s$ annihilation terms. In the following we shall discuss uncertainties related to the choice of factorization scale μ_F^2 .

The matrix elements for single-parton scattering were calculated using color-connected helicity amplitudes. They allow for an explicit exact sum over colors, while the sum over helicities can be dealt with using Monte Carlo methods. The color-connected amplitudes were calculated with an automatic program similar to HELAC [36, 37], following a recursive numerical Dyson-Schwinger approach. Phase space integration was performed with the help of KALEU [38], which automatically generates importance sampled phase space points.

III. NUMERICAL RESULTS

A. $c\bar{c}c\bar{c}$ production

In this subsection we discuss quark level cross sections. It is our aim to make a detailed comparison of SPS and DPS contributions. Let us start from single-particle c (or \bar{c}) distributions.

In all calculations of the SPS contribution the CJ12 parton distribution functions are used [39]. In Fig. 1 we compare contributions for $gg \rightarrow c\bar{c}c\bar{c}$ and $q\bar{q} \rightarrow c\bar{c}c\bar{c}$ subprocesses to distributions in rapidity and transverse momentum of one of c quark (or \bar{c} antiquark) at $\sqrt{s} = 7$ TeV. As for single $c\bar{c}$ production [33] the gluon-gluon fusion gives the contributions of almost two-orders of magnitude larger than the quark-antiquark (antiquark-quark) annihilation. The shapes are, however, rather similar.

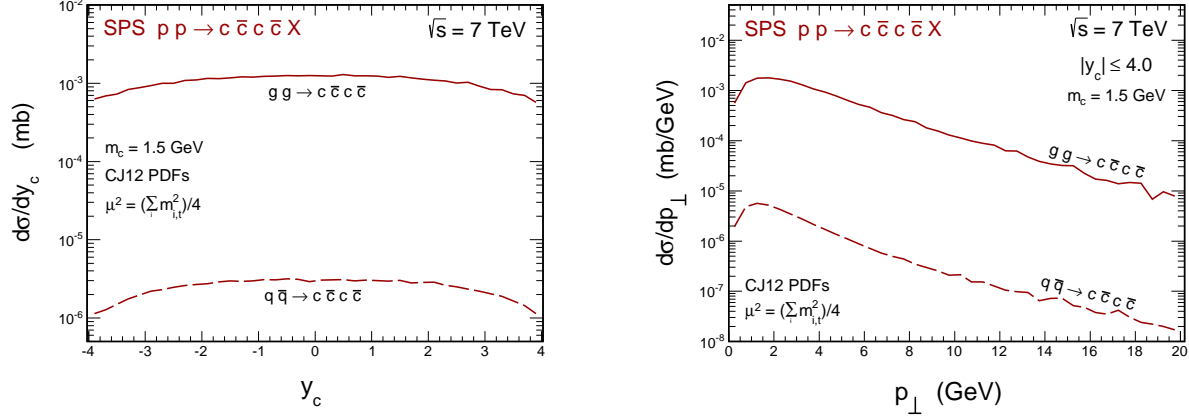


FIG. 1: Rapidity (left panel) and transverse momentum (right panel) distributions of charm quarks for $c\bar{c}c\bar{c}$ production. We compare contributions of gluon fusion and quark-antiquark annihilation.

Fig. 2 shows distributions in rapidity distance between two c quarks (or two \bar{c} antiquarks) and distribution in cc (or $\bar{c}\bar{c}$) invariant mass. The quark-antiquark component is concentrated at smaller rapidity distances and/or smaller invariant masses than the gluon-gluon component. However, in practice the quark-antiquark terms are at the LHC energies negligible and will be ignored in the following.

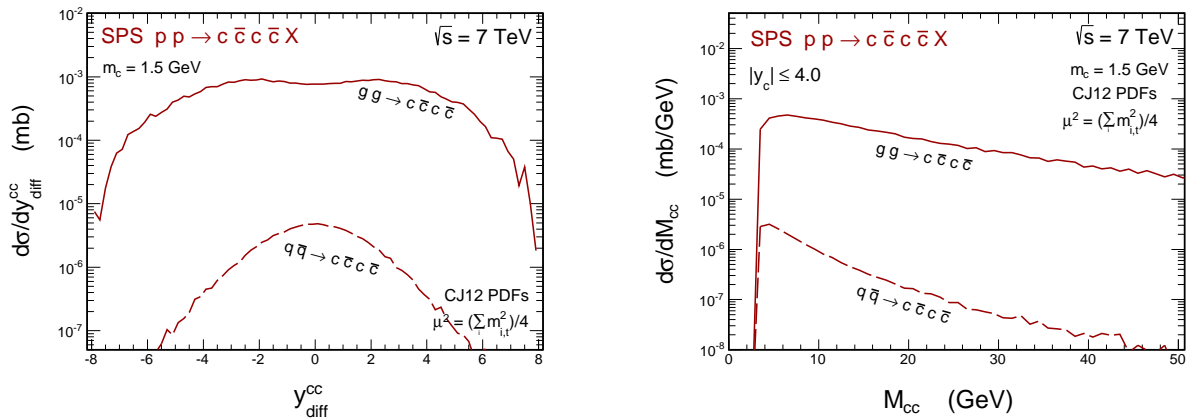


FIG. 2: Distribution in rapidity distance between two c quarks (or two \bar{c} antiquarks) and in invariant mass of cc (or $\bar{c}\bar{c}$) system.

In Fig. 3 we present again rapidity (left panel) and transverse momentum (right panel) distributions for the dominant gluon-gluon $c\bar{c}c\bar{c}$ production. The uncertainties in DPS contribution are shown as the dashed band. The new exact calculation of SPS contribution is

shown as the dashed line and compared to the approximate high-energy Szczurek-Schäfer approach [5] shown by the dash-dotted line. Here in the exact SPS calculations the factorization and renormalization scales were fixed at $\mu_R^2 = \mu_F^2 = (\sum_{i=1}^4 m_{i,\perp}^2)/4$. One can clearly see that the SPS contribution is much smaller than the DPS contribution, which confirms that the production of $c\bar{c}c\bar{c}$ is an ideal place to study double parton effects as advocated already in [1].

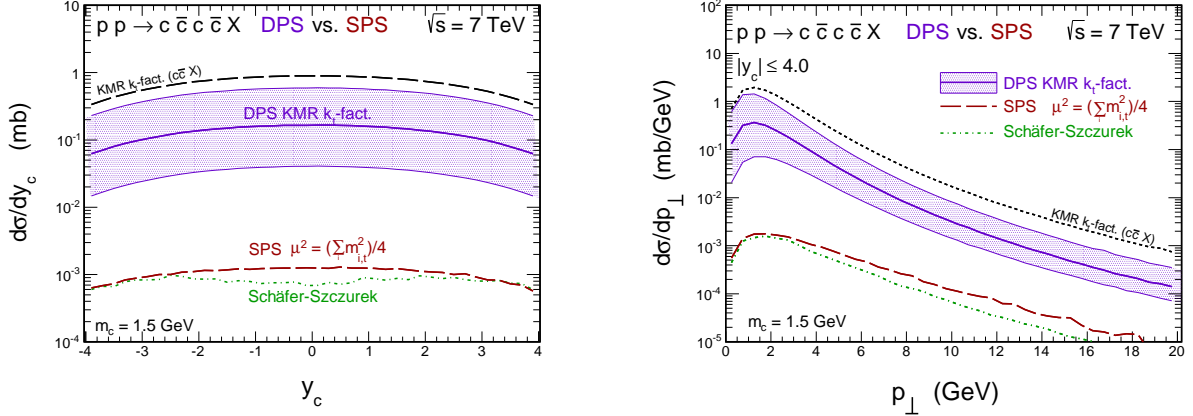


FIG. 3: Rapidity (left panel) and transverse momentum (right panel) distributions of charm quarks for $c\bar{c}c\bar{c}$ production.

The dependence on the choice of scales is quantified in Fig. 4. Uncertainties up to factor 4 can be observed. The choice $\mu_R^2 = \mu_F^2 = M_{c\bar{c}c\bar{c}}^2$ gives the smallest result.

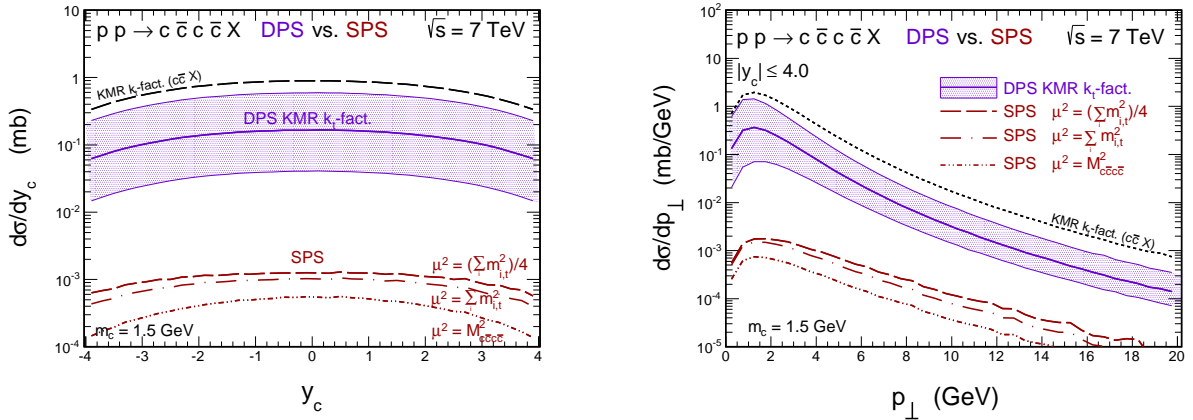


FIG. 4: Uncertainties of the SPS contribution due to the choice of renormalization and factorization scale.

In Fig. 5 we present uncertainties on the choice of charm quark mass. The related uncertainty is much smaller than that related to the choice of scales.

In Fig. 6 we show invariant mass distribution of the cc (or $c\bar{c}$) and $c\bar{c}c\bar{c}$ systems. We present separate contributions of DPS and SPS. In addition, we compare results of calculations with exact and approximate matrix elements. One can see a clear difference at small

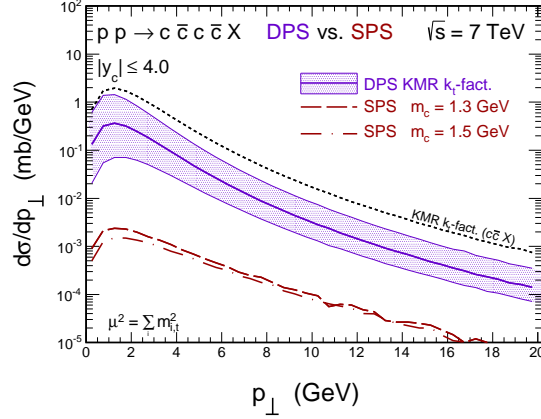


FIG. 5: Uncertainties of the SPS contribution due to the choice of charm quark mass.

invariant masses, which is due to absence of the gluon splitting mechanism in the high-energy approximation of Ref. [5].

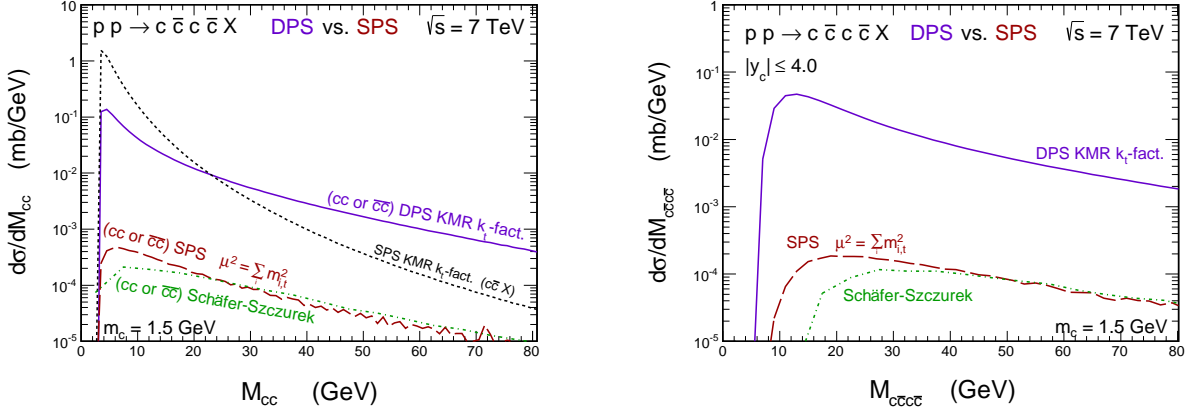


FIG. 6: Invariant mass distribution of two c quarks (or two \bar{c} antiquarks) (left panel) and of the $c\bar{c}c\bar{c}$ system (right panel) for DPS and SPS with exact and approximate matrix element.

Some one-dimensional correlation distributions are shown in Fig. 7. In the left panel we present the distribution in rapidity distance between the two c quarks (or two antiquarks) and in the right panel the distribution in relative azimuthal angle between the two quarks (or two antiquarks). Both in DPS and SPS large rapidity distances (between cc or $\bar{c}\bar{c}$) are present. On the other hand, the distributions in relative azimuthal angle are qualitatively different. While the DPS contribution is completely flat, the SPS contribution is rising towards the back-to-back configuration where it takes a maximum. Since the SPS contribution is much smaller the rise cannot be observed in real experiments. Both the approximate (Schäfer-Szczurek) and the exact result give very similar dependence in relative azimuthal angle. For comparison we show also distribution between c and \bar{c} in SPS production of one $c\bar{c}$ pair calculated in the k_t -factorization approach with KMR UGDF.

Particularly interesting are correlations in rapidities of two c quarks. In Fig. 8 we compare results obtained for DPS (left panel) and SPS (right panel). In our DPS model the two c quarks are completely decorrelated. In the SPS calculation the two quarks are correlated

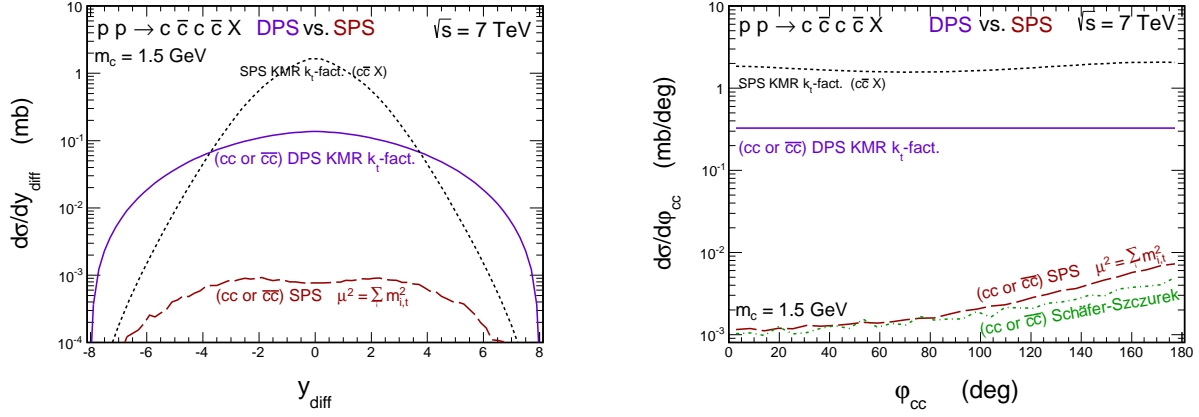


FIG. 7: Distribution in the rapidity distance between two c quarks (or two \bar{c} antiquarks) (left panel) and distribution in relative azimuthal angle between two c quarks (or two \bar{c} antiquarks) (right panel).

via respective matrix element and energy-momentum conservation.

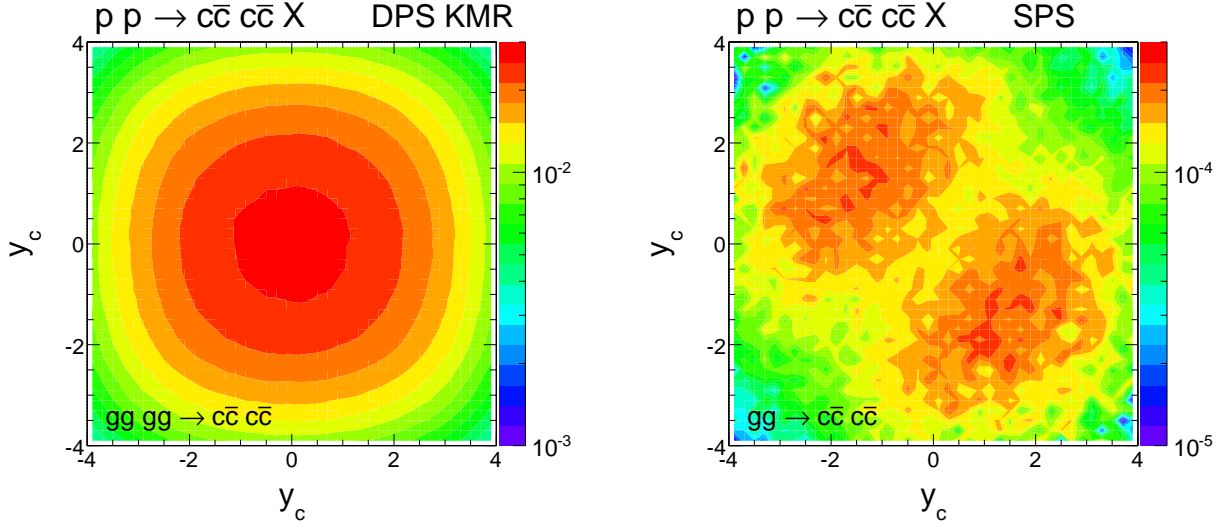


FIG. 8: Correlation in rapidities between two c quarks from the $c\bar{c}c\bar{c}$ event for DPS (left panel) and SPS (right panel).

In the next section we shall present results at the hadron level.

B. D meson correlations

Now we wish to present our results for D mesons. Below we shall consider only D^0 meson production studied recently by the LHCb collaboration.

In Fig. 9 we show distributions in relative rapidity distance between two D^0 mesons with kinematical cuts (rapidities and transverse momenta) corresponding to the LHCb experiment. The distribution normalized to the total cross section is shown in the right panel.

The shape of the distribution is well reproduced.

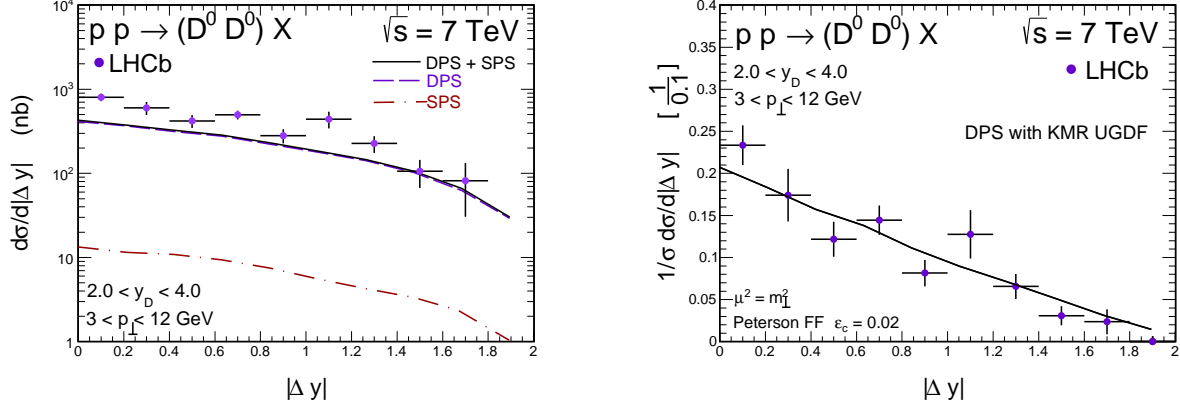


FIG. 9: Distribution in rapidity difference between two D^0 mesons (left panel). The SPS contribution (dash-dotted line) is compared to the DPS contribution (dashed line). The right panel shows distribution normalized to the total cross section.

In Fig. 10 we compare the distribution in rapidity distance between D^0 and D^0 to a similar distribution for the distance between D^0 and \bar{D}^0 . The latter distribution falls down somewhat faster. This is shown both in logarithmic (left panel) and linear (right panel) scale.

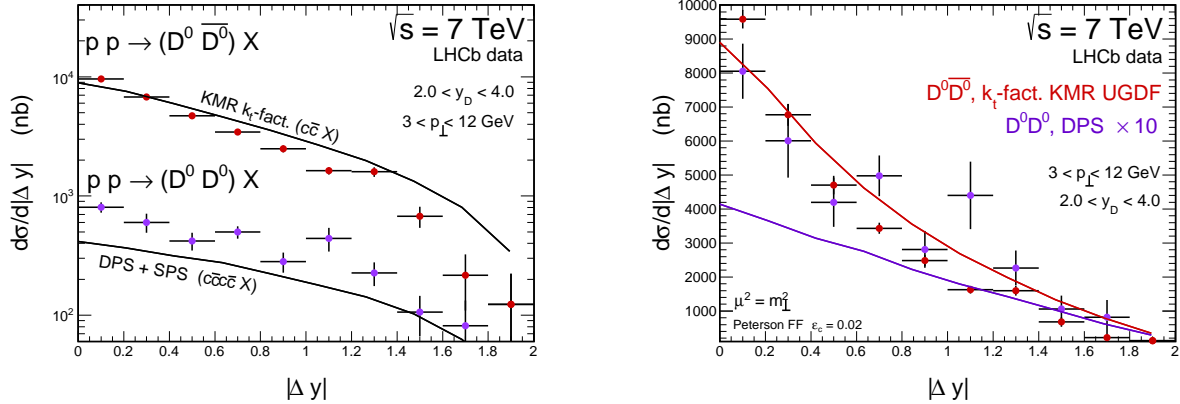


FIG. 10: Distribution in rapidity distance between two D^0 mesons and between $D^0 \bar{D}^0$ from SPS production.

Other distributions in meson transverse momentum and two-meson invariant mass are shown in Fig. 11. The shape in the transverse momentum is almost correct but some cross section is lacking. Two-meson invariant mass distribution is shown in the right panel. One can see some lacking strength at large invariant masses.

We close our presentation with azimuthal angle correlation. In Fig. 12 we compare correlations for $D^0 D^0$ and $D^0 \bar{D}^0$. The distribution for identical mesons is somewhat flatter than that for $D^0 \bar{D}^0$ which is consistent with the dominance of the DPS contribution.

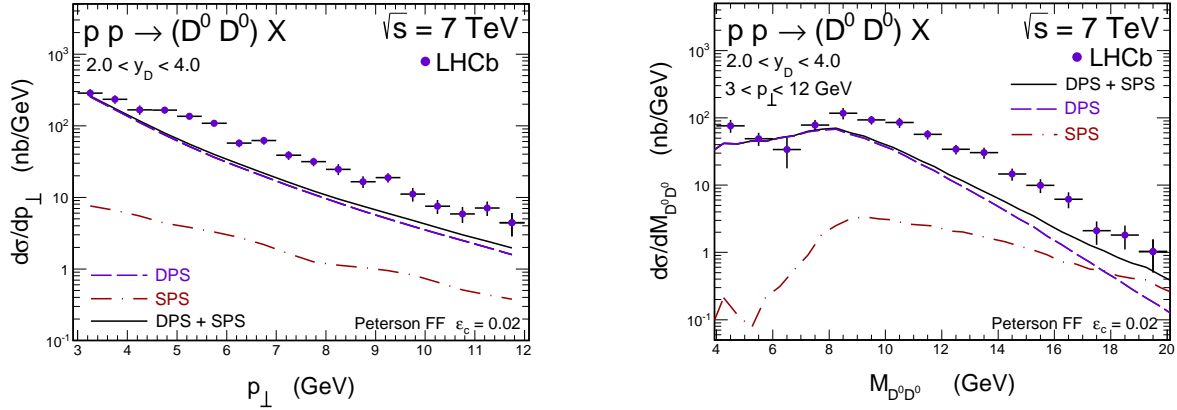


FIG. 11: Distributions in meson transverse momentum when both mesons are measured within the LHCb acceptance and corresponding distribution in meson invariant mass for DPS and SPS contributions.

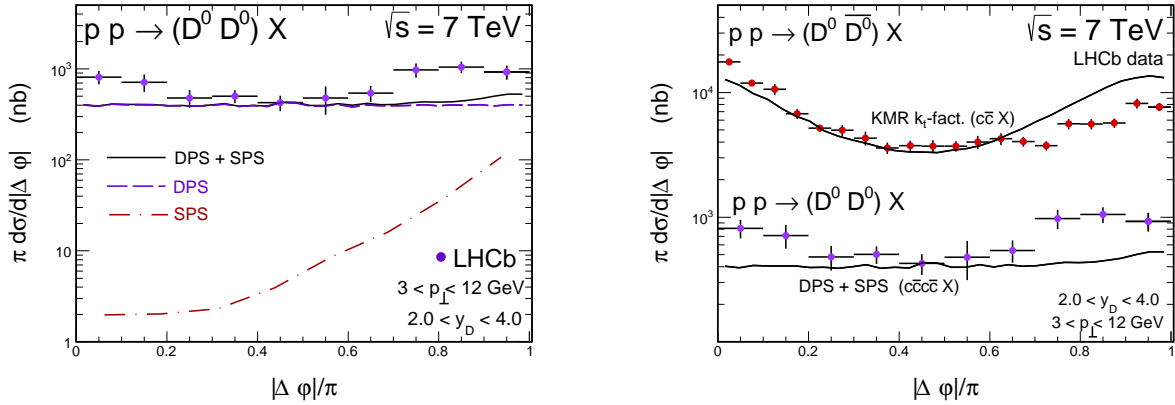


FIG. 12: Azimuthal angle correlation between $D^0 D^0$ and $D^0 \bar{D}^0$.

IV. CONCLUSIONS

In the present analysis we have compared several distributions for double charm production for double parton and single parton scattering. In the latter case we have performed the calculation with exact matrix element for both $gg \rightarrow c\bar{c}c\bar{c}$ and $q\bar{q} \rightarrow c\bar{c}c\bar{c}$ subprocesses (all possible leading-order diagrams).

First, we have discussed results at the quark level. Both one particle (rapidity, transverse momentum) and correlation (invariant masses, rapidity distances) distributions have been presented. We have shown that the results of single parton scattering with exact and approximate (previously used in the literature) matrix element differ only in some corners of the phase space, in particular for small invariant masses of cc and $c\bar{c}c\bar{c}$ systems. The difference can be understood as due to gluon splitting mechanisms not present in high-energy approximation [5]. The general situation stays, however, unchanged. At the LHC energy $\sqrt{s} = 7$ TeV one observes an unprecedented dominance of double parton scattering. At the nominal energy $\sqrt{s} = 14$ TeV the dominance would be even larger. This opens a possibility

to study the DPS effects in the production of charmed mesons.

In the next stage we have performed a calculation for production of D mesons. Here we have chosen kinematics relevant for the LHCb experiment. In contrast to ATLAS or CMS, the LHCb apparatus can measure only rather forward mesons but down to very small transverse momenta. We have calculated several distributions and compared our results with experimental data of the LHCb collaboration. Our DPS mechanism gives a reasonable explanation of the measured distribution. Some strength is still missing. This can be due to $3 \rightarrow 4$ processes discussed e.g. in Ref. [40] in the context of four jet production. This will be a subject of separate studies.

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